## SKEW SHOCK WAVE ON A GAS-SOLID BODY INTERFACE

S. K. Andilevko

UDC 534.2

The interaction between a skew shock wave (SSW) and a plane surface of a solid body is considered. Break decay (BD) on the gas-solid body interface at an arbitrary angle of impact is calculated within the limits of applicability of the hydrodynamic theory of shock waves.

The entire set of possible angles of interaction between an SSW and a surface of a solid body (SSB) within the limits of applicability of the hydrodynamic theory of shock waves [1] can be divided conventionally into two regions: 1) a region of reflection of shock waves (SW) from a rigid surface (RS); 2) a region where the SSB cannot be considered to be an RS and BD on it involves not only the SSW and a reflected shock wave (RISW) but also a refracted shock wave (RrSW) in the solid body.

Of course theoretically, for any velocity $D$ of the SSW one can indicate an angle of impact with the SSB $\varphi$ at which the velocity $q_{1}$ of the contact point (CP) $O$ (Fig. 1) exceeds the velocity of sound $c_{s}$ in the solid body (SB). However, since the processes of regular interaction for rigid and compressible surfaces are qualitatively similar, it would be logical to divide the two regions of interaction by means of the angle of impact $\theta_{c}$ at which the regime of reflection on the rigid surface changes [2]. Then, SSWs with $D \leq c_{s} \sin \theta_{c}$ refer to the first region, and the rest refer to the second region. First, we turn our attention to consideration of BD on the SSB in the second region, where $D$ is assuredly higher than $c_{s} \sin \theta_{c}$.

We define all the parameters in regions (media) 1 and 5 (the original gas and SB) in a coordinate system connected to the CP as $\rho_{1}, p_{1}, q_{1}=D / \sin \varphi$ and $\rho_{5}=\rho_{\mathrm{s}}, p_{\mathrm{s}}=p_{1}, q_{\mathrm{s}}=q_{1}=D / \sin \varphi$, respectively (Fig. 1). Considering the SW in the SB in the hydrodynamic approximation, it is quite admissible [1, 3] to use the Tate equation

$$
\begin{equation*}
p_{4}=a\left[\left(\frac{\rho_{4}}{\rho_{5}}\right)^{m}-1\right], \tag{1}
\end{equation*}
$$

where $a=\rho_{\mathrm{s}} c_{\mathrm{s}}^{2} / m, m=4$ [3], as the equation of state of the SB, although it should be noted that the form of the equation of state is more specified for definiteness. The gas is specified by a polytrope with the constant index $k$. The parameters of the gas behind the SSW (medium 2 in Fig. 1) can be determined by means of the Rankine-Hugoniot relation and the equation of the polytrope:

$$
\begin{gather*}
p_{2}=\frac{2 \rho_{1} q_{1}^{2}}{k+1} \sin ^{2} \varphi-\frac{k-1}{k+1} p_{1}, \frac{\rho_{1}}{\rho_{2}}=K\left(p_{2} / p_{1}\right)=\frac{k+1+(k-1) p_{2} / p_{1}}{k-1+(k+1) p_{2} / p_{1}}, \\
u_{2}=q_{1} \sin \varphi K\left(p_{2} / p_{1}\right), \quad \theta_{2}=\varphi-\arccos \sqrt{\left(\frac{1}{1+\tan ^{2} \varphi K\left(p_{2} / p_{1}\right)^{2}}\right),}  \tag{2}\\
q_{2}=\sqrt{q_{1}^{2} \cos ^{2} \varphi+q_{1}^{2} \sin ^{2} \varphi K\left(p_{2} / p_{1}\right)^{2}}=q_{1} \cos \varphi \sqrt{1+\tan ^{2} \varphi K\left(p_{2} / p_{1}\right)^{2}}
\end{gather*}
$$

for medium 3:

Scientific-Research Institute of Pulse Processes of the Belarusian State Scientific and Production Concern of Powder Metallurgy, Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 72, No. 2, pp. 218227, March-April, 1999. Original article submitted April 3, 1998.


Fig. 1. Regular reflection of an SSW.

$$
\begin{gather*}
p_{3}=\frac{2 \rho_{2} q_{2}^{2}}{k+1} \sin ^{2}\left(\beta+\theta_{2}\right)-\frac{k-1}{k+1} p_{2}, u_{3}=q_{2} \sin \left(\beta+\theta_{2}\right) K\left(p_{3} / p_{2}\right), \\
\rho_{3} / \rho_{2}=K\left(p_{3} / p_{2}\right)^{-1}, \theta_{3}=\arccos \sqrt{\left(\frac{1}{1+\tan ^{2}\left(\beta+\theta_{2}\right) K\left(p_{3} / p_{2}\right)^{2}}\right)-\beta}, \\
q_{3}=q_{2} \cos \left(\beta+\theta_{2}\right) \sqrt{1+\tan ^{2}\left(\beta+\theta_{2}\right) K\left(p_{3} / p_{2}\right)^{2}} \tag{3}
\end{gather*}
$$

for medium 4:

$$
\begin{gather*}
p_{4}=p_{1}+\rho_{\mathrm{s}} q_{1}^{2} \sin ^{2} \alpha\left[1-\left(1+\frac{p_{4}}{a}\right)^{-1 / m}\right], u_{4}=q_{1} \sin \alpha\left(1+\frac{p_{4}}{a}\right)^{-1 / m} \\
\frac{\rho_{4}}{\rho_{5}}=\left(1+\frac{p_{4}}{a}\right)^{1 / m}, \theta_{4}=\alpha-\arccos \sqrt{ }\left(\frac{1}{1+\tan ^{2} \alpha\left(1+\frac{p_{4}}{a}\right)^{-2 / m}}\right)  \tag{4}\\
q_{4}=q_{1} \cos \alpha \sqrt{\left(1+\tan ^{2} \alpha\left(1+\frac{p_{4}}{a}\right)^{-2 / m}\right)}
\end{gather*}
$$

To solve it simultaneously, the total system of equations (2)-(4) yields 15 equalities for determination of 17 unknown quantities: $p_{2}, p_{3}, p_{4}, \rho_{2}, \rho_{3}, \rho_{4}, q_{2}, q_{3}, q_{4}, u_{2}, u_{3}, u_{4}, \theta_{2}, \theta_{3}, \theta_{4}, \beta$, and $\alpha$. The conditions of equality of the pressure and the normal components of the mass velocity on each side of the SSB

$$
\begin{equation*}
p_{4}=p_{3}, q_{3} \sin \theta_{3}=q_{4} \sin \theta_{4} . \tag{5}
\end{equation*}
$$

close this system. The first equation of (5) allows one to relate $\alpha$ and $\beta$ :

$$
\begin{equation*}
\left.\alpha=\arcsin \sqrt{ } \frac{\frac{2}{k+1} \frac{\rho_{2}}{\rho_{\mathrm{s}}}\left(\frac{q_{2}}{q_{1}}\right)^{2} \sin ^{2}\left(\beta+\theta_{2}\right)-\frac{k-1}{k+1} \frac{p_{2}}{\rho_{\mathrm{s}} q_{1}^{2}}-\frac{p_{1}}{\rho_{\mathrm{s}} q_{1}^{2}}}{1-\left[1+\frac{2 \rho_{2} q_{2}^{2}}{(k+1) a} \sin ^{2}\left(\beta+\theta_{2}\right)-\frac{k-1}{k+1} \frac{p_{2}}{a}\right]^{-1 / m}}\right) . \tag{6}
\end{equation*}
$$

We note that $p_{s}, q_{1}, a, k$, and $p_{1}$ are the initial parameters of the problem, and $q_{2}, \rho_{2}, \theta_{2}$, and $p_{2}$ are uniquely determined in terms of them in (2). The second equation of (5) leads to the transendental relation

$$
\begin{align*}
& q_{2} \cos \left(\beta+\theta_{2}\right) \sqrt{1+\tan ^{2}\left(\beta+\theta_{2}\right) K\left(p_{3} / p_{2}\right)^{2}} \sin \left(\arccos \sqrt{\left.\left(\frac{1}{1+\tan ^{2}\left(\beta+\theta_{2}\right) K\left(p_{3} / p_{2}\right)^{2}}\right)-\beta\right)=}\right. \\
& \quad=q_{1} \cos \alpha \sqrt{ }\left(1+\tan ^{2} \alpha\left(1+\frac{p_{3}}{a}\right)^{-2 / m}\right) \sin \left(\alpha-\arccos \sqrt{\left.\left(\frac{1}{1+\tan ^{2} \alpha\left(1+\frac{p_{3}}{a}\right)^{-2 / m}}\right)\right)} .\right. \tag{7}
\end{align*}
$$

from which $\beta$ is calculated using (6), and then all the parameters of the problem are calculated by means of (2)-(4). The angle of rotation of the SSB is

$$
\begin{equation*}
\lambda=\arctan \frac{q_{4} \sin \theta_{4}}{q_{1}} . \tag{8}
\end{equation*}
$$

However, before passing to a direct analysis of the obtained solution, one should determine two angles that are very important for subsequent considerations. In the interaction between the SSW and the SSB, for any $D$ there is an angle $\varphi_{c}$ after which the flow behind the SSW becomes subsonic. This angle is determined by the condition

$$
\begin{equation*}
\frac{q_{2}}{c_{2}} \geq 1 \text { or } \operatorname{ctan} \varphi \geq \sqrt{\left(\frac{2 k K\left(p_{2} / p_{1}\right)}{k+1}-\frac{k-1}{k+1} \frac{k p_{1}}{\rho_{1} D^{2}} K\left(p_{2} / p_{1}\right)-K\left(p_{2} / p_{1}\right)^{2}\right) . . ~ . ~ . ~} \tag{9}
\end{equation*}
$$

An SW for which the SSB cannot be treated as an RS is very strong for gases: $p_{2} \gg p_{1}, D \gg c_{1}$, and $K\left(p_{2} / p_{1}\right) \approx(k-1) /(k+1)$. Therefore, instead of (9) we obtain

$$
\begin{equation*}
\varphi \leq \varphi_{c}=\operatorname{arcctan} \sqrt{\left(\frac{k-1}{k+1}\right) .} \tag{10}
\end{equation*}
$$

For air $\left.\varphi_{1}=1.29 \mathrm{~kg} / \mathrm{m}^{3} ; k=1.4\right) \varphi_{c} \approx 67.7^{\circ}$. For $\varphi \geq \varphi_{c}$ BD on the SSB can no longer be calculated by means of (2)-(4), because the subsonic character of the flow behind the SSW does not allow the appearance of an RISW. The second angle that is characteristic of the given problem is defined by the existence of a special regime in which complete refraction of the SSW takes place (similar to the angle of change in the regime of interaction for gases [4, 5 ]). This angle can be determined in simultaneous solution of systems (2) and (4) with $\varphi$ replaced by $\varphi_{1}$. The system contains 10 equations for determination of 12 unknown quantities: $p_{2}, p_{4}, \rho_{2}, \rho_{4}, q_{2}, q_{4}, u_{2}, u_{4}, \theta_{2}, \theta_{4}, \varphi_{1}$, and $\alpha$. The system is closed by the conditions of equality of the pressure and the normal velocity of the flow on each side of the SSB:

$$
\begin{equation*}
p_{2}=p_{4}, q_{2} \sin \theta_{2}=q_{4} \sin \theta_{4} . \tag{11}
\end{equation*}
$$

The first equation of (11) allows one to find a unique relation between $\alpha$ and $\varphi_{t}$ :

$$
\begin{equation*}
\left.\sin \alpha=\sin \varphi \sqrt{ } \frac{\frac{2}{k+1} \frac{\rho_{1}}{\rho_{\mathrm{s}}}-\frac{2 k}{k+1} \frac{p_{1}}{\rho_{\mathrm{s}} D^{2}}}{1-\left[1+\frac{2 \rho_{1} D^{2}}{(k+1) a}-\frac{k-1}{k+1} \frac{p_{1}}{a}\right]^{-1 / m}}\right), \tag{12}
\end{equation*}
$$

and the transcendental equation for calculation of $\varphi_{t}$

TABLE 1. Values of the Angle of Total Refraction for Certain Metals

| $D, \mathrm{~m} / \mathrm{sec}$ | Angle of total refraction $\varphi_{\mathrm{l}}$, deg |  |  |
| :---: | :---: | :---: | :---: |
|  | aluminum | iron | lead |
| 6200 | 89.99036 | 89.99687 | 89.98346 |
| 8000 | 89.98315 | 89.99446 | 89.97687 |
| 10,000 | 89.97665 | 89.99214 | 89.97304 |




Fig. 2. Curves $\beta(\varphi)\left(\varphi \leq \varphi_{\mathrm{c}}\right)$ (a) and $\alpha(\varphi)$ (b) [1) $D=6000 \mathrm{~m} / \mathrm{sec}, 2$ 2) 800 , 3) $10,000 \mathrm{~J}$.

$$
\begin{gather*}
\cos \varphi_{\mathrm{t}} \sqrt{1+\tan ^{2} \varphi_{\mathrm{t}} K\left(p_{2} / p_{1}\right)^{2}} \sin \left(\phi_{\mathrm{t}}-\arccos \sqrt{\left.\left(\frac{1}{1+\tan ^{2} \varphi_{\mathrm{t}} K\left(p_{2}^{\prime} / p_{1}\right)^{2}}\right)\right)=}\right. \\
=\cos \alpha \sqrt{\left(1+\tan ^{2} \alpha\left(1+\frac{p_{2}}{a}\right)^{-2 / m}\right) \sin \left(\alpha-\arccos \sqrt{\left(\frac{1}{1+\tan ^{2} \alpha\left(1+\frac{p_{2}}{a}\right)^{-2 / m}}\right)}\right) .} . \tag{13}
\end{gather*}
$$

follows from the second equation of (11). Values of $\varphi_{t}$ for several metals for various $D$ are presented in Table 1. The value of $\varphi_{t}$ is close to $\pi / 2$ for all metals and always exceeds $\varphi_{c}$ for gases. As is seen from Table 1 , the angle $\varphi_{t}$ increases with an increase in the acoustic impedance of the SB.

Analyzing the solution of (2)-(4) for $\varphi_{1}>\varphi_{c}$, we note that a regime of regular reflection exists for all $\varphi<\varphi_{\mathrm{c}}$. Dependences of the angle of reflection $\beta$ and the angle of refraction $\alpha$ on $\varphi$ for different metals for various $D$ are presented in Fig. 2a and b, respectively, with $\alpha(\varphi)$ (Fig. 2b) being presented for the entire set of angles $\varphi$, including the regime of irregular interaction. In shape, $\beta(\varphi)$ is a very stable loop whose parameters are slightly related to the nature of the metal, and therefore, Fig. 2a gives this curve for just one metal because all the rest would virtually merge with it. The characteristics of this loop are much more strongly affected by the parameters of the gas above the SSB.

As soon as $\varphi \geq \varphi_{c}$, the RISW disappears, and there is no simultaneous solution of (2)-(4). There are two mutually exclusive possibilities for solving the problem arising: 1) related to the assumption that at the CP a rarefaction wave (RW) departs from the SSB for the same reason that was used to solve a similar problem in gases [4];2) based on the assumption that a turn of the vector of the gas flow not compensated by the RISW leads to separation of the point $O$ from the SSB and realization of an irregular regime of reflection similar to that occurring on a rigid surface and first described by E. Mach.

The first assumption has already been used and is not devoid of certain defects: a) the angle $\varphi_{\mathrm{t}}$ at which total refraction of the SSW is realized, though being large for metals, is still less than $\pi / 2$, and for the case of an RW it is by no means related to the solution in spite of the fact that here the RW should disappear (and immediately appear as soon as $\varphi \neq \varphi_{1}$ ); b) the introduction of an RW to the considerations in no way agrees with the well-known $[1,2]$ solution for SSW reflection from a rigid surface, repeatedly confirmed by experiment [6, 7]. The possibility of natural passage to a rigid surface is simply absent in this approach.


Fig. 3. Scheme of break decay in the case of irregular interaction, $\varphi_{t}>\varphi>\varphi_{c}$.

By virtue of this we consider irregular interaction in more detail. Since for the majority of metals $\varphi_{t}>\varphi_{c}$, and irregular reflection begins from the moment when $\varphi>\varphi_{c}$, strong Mach reflection with an RISW will not be observed. The Mach wave (Fig. 3) connecting the points $O$ and $O^{\prime}$ is, generally speaking, curvilinear (judging by the experimental data of $[6,7]$ it is a mildly sloping curve). Any curve can be divided continuously into small rectilinear segments, with the higher the accuracy the more segments in the division. The sections of the wave directly adjacent to the points $O$ and $O^{\prime}$ will already be rectilinear, and the angles of their slope to the horizontal can be found on the basis of the hydrodynamic theory of SWs. A successive change in the angle of slope of the sections approximating the curve of the Mach wave (MW) allows one to calculate the change in the parameters of the SW along it and to describe the entire configuration. For this purpose it is sufficient to find the angles of incidence of the MW at the points $O$ and $O^{\prime}$, because then a smooth variation of thẹ flow charactersitics can be approximated by an arc of a circle connecting these angles [5].

The angle of incidence at the point $O$ (Fig. 3) is determined by simultaneous solution of (4) and

$$
\begin{gather*}
p_{\mathrm{m}}=\frac{2 \rho_{1} q_{1}^{2}}{k+1} \sin ^{2} \mu-\frac{k-1}{k+1} p_{1}, u_{\mathrm{m}}=q_{1} \sin \mu K\left(p_{\mathrm{m}} / p_{1}\right), \\
\rho_{\mathrm{m}} / \rho_{1}=K\left(p_{\mathrm{m}} / p_{1}\right)^{-1}, \theta_{\mathrm{m}}=\mu-\arccos \sqrt{\left(\frac{1}{1+\tan ^{2} \mu K\left(p_{\mathrm{m}} / p_{1}\right)^{2}}\right),}  \tag{14}\\
q_{\mathrm{m}}=q_{1} \cos \mu \sqrt{1+\tan ^{2} \mu K\left(p_{\mathrm{m}} / p_{1}\right)^{2}}
\end{gather*}
$$

with the conditions on each side of the SSB

$$
\begin{equation*}
p_{\mathrm{m}}=p_{4}, q_{4} \sin \theta_{4}=q_{\mathrm{m}} \sin \theta_{\mathrm{m}} \tag{15}
\end{equation*}
$$

Equations (15) establish a relation between $\alpha$ and $\mu$ and make it possible to obtain a transcendental relation for calculation of $\mu$ :

$$
\begin{equation*}
\alpha=\arcsin \sqrt{ }\left(\frac{\frac{2}{k+1} \frac{\rho_{2}}{\rho_{\mathrm{s}}} \sin ^{2} \mu-\frac{2 k}{k+1}-\frac{p_{1}}{\rho_{\mathrm{s}} q_{1}^{2}}}{1-\left[1+\frac{2 \rho_{2} q_{2}^{2}}{(k+1) a} \sin ^{2} \mu-\frac{k-1}{k+1} \frac{p_{1}}{a}\right]^{-1 / m}}\right), \tag{16}
\end{equation*}
$$



Fig. 4. Curves $\mu(\varphi)$ for iron (a), lead (b), aluminum at $D=6000 \mathrm{~m} / \mathrm{sec}$ (c), and aluminum at $D=10,000 \mathrm{~m} / \mathrm{sec}$ (d).

$$
\begin{gather*}
\cos \mu \sqrt{1+\tan ^{2} \mu K\left(p_{\mathrm{m}} / p_{1}\right)^{2}} \sin \left(\mu-\arccos \sqrt{\left(\frac{1}{1+\tan ^{2} \mu K\left(p_{\mathrm{m}} / p_{1}\right)^{2}}\right)}\right)= \\
=\cos \alpha \sqrt{\left(1+\tan ^{2} \alpha\left(1+\frac{p_{\mathrm{m}}}{a}\right)^{-2 / m}\right) \sin \left(\alpha-\arccos \sqrt{\left(\frac{1}{1+\tan ^{2} \alpha\left(1+\frac{p_{\mathrm{m}}}{a}\right)^{-2 / m}}\right)} .\right.} . \tag{17}
\end{gather*}
$$

There is no need to calculate the angle of incidence of the MW at the point $O^{\prime}$, because for all cases of irregular reflection without an RISW it coincides with $\varphi$ (otherwise a discontinuous solution, which is impossible under the given conditions, would arise). Thus, irregular reflection is fully determined here by simultaneous solution of systems (4), (14) with conditions (16), (17). Results of calculations when air was taken as the gas and various metals (iron, lead, aluminum) were taken as the SB are presented in the form of the curves $\mu(\varphi)$ (Fig. 4a-d) and $\alpha(\varphi)$ (see Fig. 2b) when $\varphi \geq \varphi_{c}$. Their analysis shows that as long as $\varphi_{t}>\varphi>\varphi_{c}, \mu>\varphi$ the weak regime of irregular reflection (WRIR), called so in [5], is realized when the MW convexity is turned to the left, the point of the center of the approximating sector lies to the right of the CP below the SSB $(\mu<\pi / 2)$, and $\boldsymbol{v}=\mu-\varphi$ (Fig. 5 b ) [5]. As soon as $\varphi=\varphi_{t}$, the regime of total reflection is realized (the only case where the MW is straight and coincides with the SSW line); in this case a change in the orientation of the MW curvature takes place. For all $\varphi>\varphi_{\mathrm{t}}$, the angle $\mu<\varphi$, which, according to [5], corresponds to the regime of strong shockless irregular reflection (SSIR) (Fig. 5a), when the MW convexity is turned to the right, the point of the center of curvature lies to the left of the CP above the plane of the tangential break (TB), and the apex angle of the sector based on the arc $O O^{\prime}$ is $\boldsymbol{v}=\varphi-\mu$ (Fig. 5b). The angle of slope of the TB for both orientations of the MW is

$$
\begin{equation*}
\tau=\arctan \frac{q_{2} \sin \theta_{2}}{q_{1}} \tag{18}
\end{equation*}
$$

As has already been noted, for virtually all metals, in the range of values of $D$ where the properties of the surface should be taken into account, the strong regime of irregular reflection (SIR), [5] with an RISW is impossible, because $\varphi_{\mathrm{t}}>\varphi_{\mathrm{c}}$. For metals this regime is realized only in the range of values of $D$ where the SSB can be considered rigid. However, when an SSW reaches a contact surface of the type gas-gas, gas-porous or powder body, and liquid-liquid or SB-SB (where in the majority of cases the flow will be supersonic at virtually any $\varphi$ due to low compressibility) this regime becomes possible if $\varphi_{\mathrm{t}}<\varphi<\varphi_{\mathrm{c}}$ (Fig. 6a). The value of the angle of incidence


Fig. 5. Schemes of break decay in the case of irregular interaction and $\varphi>\varphi_{\mathrm{t}}>\varphi_{\mathrm{c}}$ for determination of the flow behind the fronts of the SW (a) and the approximating sector of the curve (b).
of the MW at the point $O$ found earlier remains the same, but it should be supplemented by the value of the angle of incidence of the MW at the triple point $O^{\prime}$. The solution for the triple point in gases is already available [2, 8], and it was also given in [5], but for completeness of presentation we give the method of obtaining it. For this purpose, we need to calculate simultaneously systems (2), (3) and

$$
\begin{gather*}
p_{\nu}=\frac{2 \rho_{1} q_{1}^{2}}{k+1} \sin ^{2} v-\frac{k-1}{k+1} p_{1}, u_{\nu}=q_{1} \sin \nu K\left(p_{\nu} / p_{1}\right), \\
\rho_{\nu} / \rho_{1}=K\left(p_{\nu} / p_{1}\right)^{-1}, \theta_{\nu}=v-\arccos \sqrt{\left(\frac{1}{1+\tan ^{2} \nu K\left(p_{\nu} / p_{1}\right)^{2}}\right),}  \tag{19}\\
q_{\nu}=q_{1} \cos v \sqrt{1+\tan ^{2} v K\left(p_{\nu} / p_{1}\right)^{2}}
\end{gather*}
$$

with the conditions on each side of the TB

$$
\begin{equation*}
p_{\nu}=p_{3}, q_{3} \sin \theta_{3}=q_{\nu} \sin \theta_{\nu}, \tag{20}
\end{equation*}
$$

which lead, respectively, to the equations

$$
\begin{gather*}
\sin v=\sqrt{\left(\frac{\rho_{2}}{\rho_{1}}\left(\frac{q_{2}}{q_{1}}\right)^{2} \sin ^{2}\left(\beta+\theta_{2}\right)-\frac{k-1}{2} \frac{p_{2}-p_{1}}{\rho_{1} q_{1}^{2}}\right),}  \tag{21}\\
q_{2} \cos \left(\beta+\theta_{2}\right) \sqrt{1+\tan ^{2}\left(\beta+\theta_{2}\right) K\left(p_{3} / p_{2}\right)^{2}} \times \\
\times \sin \left(\arccos \sqrt{\left.\left(\frac{1}{1+\tan ^{2}\left(\beta+\theta_{2}\right) K\left(p_{3} / p_{2}\right)^{2}}\right)-\beta\right)=}\right. \\
\left.=q_{1} \cos v \sqrt{1+\tan ^{2} v K\left(p_{\nu} / p_{1}\right)^{2}} \sin \left(v-\arccos \sqrt{\left(\frac{1}{1+\tan ^{2} v K\left(p_{\nu} / p_{1}\right)}\right.}\right)\right), \tag{22}
\end{gather*}
$$



Fig. 6. Schemes of BD (a) and the MW (b) for the case of $\varphi_{t}<\varphi<\varphi_{c}$; of the evolutions of reflection at $\varphi_{t}<\varphi_{c}$ (c) and $\varphi_{t} \geq \varphi_{c}$ (d): regions of: 1) regular interaction, 2) strong (c) or weak (d) reflection; 3) strong shockless reflection, 4) reflected waves, 5) refracted waves.
relating $\beta$ and $\nu$ and allowing one to calculate $\beta$ and, then, all the parameters of the waves at the triple point. Calculations made for certain materials show that in this case $\pi / 2>\nu>\mu$ always. The convexity of the MW curve is turned to the right, and the point of the center of curvature, as in the case of the SSIR, lies to the left of $O$ and above the TB ( $\nu<\pi / 2$ ) (Fig. 6b). The apex angle of the cone $O F O^{\prime}$ is $v=\nu-\mu$. The most common evolutions of the MW in irregular reflection are presented graphically in Fig. 6c and d.

The data of Table 1 allow one to draw the conclusion that the value of $\varphi_{t}$ increases with increase in the acoustic impedance. Mathematically, the requirement of a rigid surface means that the impedance of the RS is infinite. Thus, passing to an analysis of the region of interaction between an SSW and an RS, we note that for all values of $D$ possible for this region the right-hand side of (13) should be zero (the coefficient $a$ for the RS becomes infinitely large). There are two possible solutions of (13) $-\cos \varphi_{1}=0$ and $\sin \varphi_{1}=0$ (since $K\left(p_{2} / p_{1} \neq 1\right.$ for the SW), i.e., $\varphi_{t}=\pi / 2$ and $\varphi_{t}=0$, respectively. The solution $\varphi_{t}=0$ is physically meaningless because it means that there should be no reflected $S W$ when a plane $S W$ reaches a rigid wall, and consequently, $\varphi_{t}=\pi / 2$ (this is the only possible configuration of an SW for an RS for which a plane one-wave structure will be stable). The region of regular reflection for an RS has been studied in detail (e.g., in [2]), and therefore we consider thoroughly the processes characteristic of the region of irregular reflection. As is known [2], the requirement of rigidity, related to the condition of parallelism of the gas flow along the wall after the CP , leads to separation of the CP from the RS and to appearance of a Mach wave for angles of interaction exceeding some characteristic [2, 8] angle $\theta_{\mathbf{c}}$. The condition for calculation of $\theta_{c}$ is in no way associated with the conditions of calculation of $\varphi_{t}$ and $\varphi_{c}$. This angle appears as an additional characteristic quantity directly connected with the condition of rigidity (mathematically formulated as the requirement of parallelism of the gas flow and the RS $\theta_{3}=0$ in (3)). In the majority of cases
$\theta_{\mathrm{c}}<\varphi_{\mathrm{c}}$ for gases, and thus, for all $\theta_{\mathrm{c}}<\varphi<\varphi_{\mathrm{c}}$ a regime of strong irregular reflection should exist. The condition of parallelism of the flow to the RS leads to the requirement $\theta_{\mathrm{m}}=0$ for the system (14), which can be realized for two values of $\mu$ :

$$
\begin{equation*}
p_{\mathrm{m}}=-\frac{k-1}{k+1} p_{1}, \frac{\rho_{\mathrm{m}}}{\rho_{1}}=0, K\left(\frac{p_{\mathrm{m}}}{p_{1}}\right) \rightarrow \infty, q_{\mathrm{m}}=q_{1}, \mu=0, \theta_{\mathrm{m}}=0 \tag{23}
\end{equation*}
$$

and

$$
\begin{gather*}
p_{\mathrm{m}}=\frac{2 \rho_{1} q_{1}^{2}}{k+1}-\frac{k-1}{k+1} p_{1}, \frac{\rho_{\mathrm{m}}}{\rho_{1}}=K\left(p_{\mathrm{m}} / p_{1}\right)^{-1}, \mu=\frac{\pi}{2}, \theta_{\mathrm{m}}=0,  \tag{24}\\
q_{\mathrm{m}}=u_{\mathrm{m}}=q_{1} K\left(p_{\mathrm{m}} / p_{1}\right) .
\end{gather*}
$$

Conditions (23) are physically meaningless, and consequently, $\mu=\pi / 2$ for any $\varphi$. The angle of incidence of the MW $\nu$ at the triple point $O^{\prime}$ can be found by simultaneous solution of (2), (3), and (19) with conditions (21) and (22), and it does not exceed $\pi / 2$. Since $\mu=\varphi_{t}=\pi / 2$ and $\nu<\pi / 2$, then $\nu \leq \mu$, and the convex side of the MW will always be turned to the left (Fig. 6a) [5]. For strong irregular reflection from an RS the point of the center of curvature lies to the right of $O$ on the contact surface, and the apex angle of the sector is $v=\mu-\nu=\pi / 2-\nu$ (Fig. 6a) [5]. As soon as $\varphi$ becomes equal to or exceeds $\varphi_{c}$, the RISW disappears, $\nu$ and $\varphi$ equalize, and weak irregular reflection is realized (Fig. 6b) [5]; for this reflection the center of curvature of the MW remains to the right on the boundary of contact, the MW convexity is turned to the left ( $\varphi \leq \mu=\pi / 2$ ), and the angle $\vartheta=\pi / 2-\varphi$. As is seen, since $\varphi_{c}<\varphi_{t}=\pi / 2$ for an RS, the MW convexity is always turned to the left, and the center of curvature lies to the right of $O$ on the contact surface. The regime of strong regular reflection with the turn of the MW convexity to the right and SSIR are virtually nonrealizable for reflection of an SSW from an RS, since $\varphi_{1}=\pi / 2$ and $\varphi \leq \varphi_{1}$. The case $\varphi=\varphi_{t}$ corresponds to propagation of a glancing wave along an RS. We note that the terms "angle of total refraction" and "angle of change in the regime" used for the angle $\varphi_{t}$ in [5] and [4], respectively, describe its essence inaccurately. Rather, this angle should be called the angle of direct refraction.

Thus, it is found that in interaction between an SSW and an SSB (as in interaction between an SSW and the interfaces gas-gas [5], gas-liquid, gas-porous body, etc., since it is of fundamental importance that the impedance of the "lower" body exceed the impedance of the "upper") in change in the quantities, when the surface of the "lower" body cannot be considered rigid, the process of reflection is determined qualitatively by the angles $\varphi_{c}$ and $\varphi_{t}$. Here the form of reflection changes in the following way:

1) in all cases where $\varphi<\varphi_{t}$ and $\varphi<\varphi_{c}$, the regime of regular reflection is realized;
2) for $\varphi_{\mathrm{c}}>\varphi_{\mathrm{t}}$ the irregular regime of interaction with a curvilinear MW is characteristic of all $\varphi^{\prime}>\varphi_{\mathrm{t}}$, the MW convexity is always turned toward the incident flow (in a system of coordinates attached to $O$ ), the center of curvature lies to the left of $O$ above the TB surface, and in this case: a) if $\varphi_{\mathrm{t}}<\varphi<\varphi_{\mathrm{c}}$, a regime with a reflected shock wave and a three-wave configuration - the strong irregular regime - is realized [5]; b) if $\varphi \geq \varphi_{\mathrm{c}}$, a regime without a reflected shock wave - strong shockless irregular reflection - is realized;
3) for $\varphi_{c}<\varphi_{t}$ irregular reflection takes place for all $\varphi \geq \varphi_{c}$, no regime with a reflected shock wave exists, and in this case: a) if $\varphi_{\mathrm{c}} \leq \varphi<\varphi_{\mathrm{t}}$, the MW convexity is always turned to the side opposite the incident flow, the center of curvature lies to the right of $O$ below the SSB - weak irregular reflection; b) if $\varphi=\varphi_{t}$, a regime of complete refraction when the front of the MW fully merges with the front of the SSW is realized; c) if $\varphi>\varphi_{t}$, the MW convexity is always turned toward the incident flow, the center of curvature lies to the left of $O$ above the TB surface - SSIR.

Reflection of an SSW from an RS occurs at $\varphi_{t}=\pi / 2$, and the requirement of parallelism of the flow to the RS results in the angle of contact between the MW and the RS $\mu=\pi / 2$, the MW convexity for all $\varphi<\pi / 2$ is turned to the side opposite the incident flow, the center of curvature lies to the right of $O$ on the contact surface and, in addition, one more characteristic angle (due to the rigidity of the interface) $\theta_{c}$ appears, which determines the change
in the reflection regime from regular to irregular. In this case for all $\theta_{\mathrm{c}} \leq \varphi<\varphi_{\mathrm{c}}$ irregular reflection is accompanied by a reflected shock wave (weak shock reflection), and for $\varphi \geq \varphi_{c}$ the RISW is absent. In the case of $\theta_{c} \geq \varphi_{c}$ (characteristic of small $D$ ) irregular reflection begins as soon as $\varphi \geq \varphi_{c}$, or it occurs for all possible angles of interaction (very small $D$ ).

In conclusion the author expresses his gratitude to G. S. Romanov, V. A. Shilkin, and V. V. Selyavko for their useful and active participation in discussion of this work.

## NOTATION

$D$, shock-wave velocity; $c$, velocity of sound in the medium; $p$, pressure; $q$, total velocity of the flow in a coordinate system connected with the point of intersection of the skew shock wave with the surface; $u$, component of the gas-flow velocity normal to the front of the shock wave; $k$, polytrope index of the gas; $a, m$, coefficient and exponent in the Tate equation, respectively; $\rho$, density; $\alpha$, angle of slope of the refracted shock wave to the SSB; $\tau$, angle of slope of the reflected shock wave to the SSB; $\varphi$, angle of slope of the skew wave to the SSB; $\lambda$, angle of rotation of the SSB; $\tau$, angle of rotation of the tangential break; $\mu$, angle of incidence of the Mach wave at the point of contact of the skew shock wave with the SSB; $v$, angle of the Mach wave at the triple point; $\theta$, angle of rotation of the vector of the total velocity of the flow behind the shock-wave front; $\boldsymbol{v}$, apex angle of the sector of a circle approximating the surface of the Mach wave; $\theta_{\mathrm{c}}$, critical angle of the change in the reflection regime for a rigid surface. Subscripts: 1 , initial parameters of the gas (medium 1); 2, parameters of medium 2 (immediately behind the skew shock wave); 3, parameters of medium 3 (immediately behind the reflected shock wave); 4, parameters of medium 4 (immediately behind the refracted shock wave); 5 , $s$, initial parameters of the solid body (medium 5 ); c, critical angle at which the supersonic flow behind the skew shock wave changs to a subsonic flow; $t$, transition angle between the regimes of irregular and regular interaction (angle of total refraction); $m$, parameters of the gas in the lower part of the Mach wave at the angle $\mu ; \nu$, parameters of the gas in the upper part of the Mach wave at the angle $v$.

## REFERENCES

1. F. A. Baum, L. P. Orlenko, K. P. Stanyukovich, V. P. Chelyshev, and V. B. Shekhter, Physics of Explosion [in Russian ], Moscow (1975).
2. K. P. Stanyukovich, Unsteady Motions of Continua [in Russian ], Moscow (1955).
3. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena [in Russian ], Moscow (1966).
4. Ch. Polakek and R. I. Sieger, in: Fundamentals of Gas Dynamics (ed. by G. Emmons) [Russian translation ], Moscow (1963), pp. 446-489.
5. S. K. Andilevko, Inzh.-Fiz. Zh., 72, No. 2, 210-217 (1999),
6. L. V. Al'tshuler, S. B. Kormer, A. A. Bakanova, et al., Zh. Eksp. Teor. Fiz., 41, No. 511, 1382-1393 (1961).
7. E. A. Feoktistova, Dokl. Akad. Nauk SSSR, 136, No. 12, 1325-1328 (1961).
8. W. Bleackney and A. Taub, Rev. Mod. Phys., 21, No. 4, 584-605 (1949).
